element method. Previous investigations have demonstrated that the inplane edge restraints have a great influence on the flutter boundaries. The present finite element formulation permits a direct and exact application of the inplane end conditions for all classical boundary conditions.

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Stability Criteria of Structural Control with Systems Noncolocated Velocity Feedback

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Introduction

FEEDBACK control with a colocated sensor and actuator has been considered having stability robustness. A sensor and actuator pair at the same location is often referred to as colocated, and any dislocation of the pair is considered a noncolocated configuration. Such a definition of colocated and noncolocated feedback is the only one meaningful in structural control implementation and experimentation. Previous work identifying the same definition of noncolocated feedback includes that of Schafer and Holzach[1] and Parks and Pak.[2] Balas[3] was among the first to consider velocity feedback with a colocated sensor and actuator. In the absence of actuator dynamics, this technique is unconditionally stable. However, having the option to use noncolocated feedback is often desirable because of design flexibility. In addition, noncolocated feedback may often be required due to physical limitation and/or hardware constraint. In the study of control/structure synthesis, Thomas and Schmit[4] considered noncolocated velocity and displacement feedback to a cantilever beam system. Because of the sensor and actuator placement, they were unable to achieve stability robustness. Cannon and Rosenthal[5] applied noncolocated feedback to a flexible structure control experiment and concluded that effective noncolocated feedback is one of the major challenges to large space structure application.

In flexible structure vibration control, the structure system is normally discretized either by finite element method or by normal mode analysis. The governing equation is hence written in the form of second-order differential equations with sparse or diagonal inertia, damping, and stiffness coefficient matrices. Prediction of stability robustness by calculating the closed-loop system spectrum for a given set of system parameters, including structural modal properties, sensor/actuator placement, and control law, is straightforward but tedious for a reasonably large number of parameters. One may argue that by keeping the mass, damping, and stiffness matrices of the closed-loop system positive definite, then the asymptotic stability is guaranteed; however, such a criterion is no longer valid in the case of noncolocated feedback because these coefficient matrices can be asymmetric. Thus, the need of design guidelines for both the control law and sensor/actuator placement that guarantee an asymptotically stable system is evident. This Note investigates the stability of second-order structural control systems with noncolocated velocity feedback. Robust stability criteria are developed for structural control in the absence of sensor and actuator dynamics. Conditions are derived such that the closed-loop system is asymptotically stable with infinite gain margin. These conditions are stated in terms of the structure system parameters such as inertia, damping, and stiffness matrices; sensor/actuator placement; and feedback control gain matrix.

Velocity Feedback

In the analysis and design of structural control, the governing equation of motion of a discretized system is often written as

$$\ddot{x} + C\dot{x} + \Lambda x = Bu \tag{1}$$

where x is an $n \times 1$ generalized coordinate vector. C is a symmetric, positive definite damping matrix, denote $C > 0$. Λ is a symmetric stiffness matrix, $\Lambda > 0$ for most structure systems; however, it can also be a negative definite, $\Lambda < 0$, in stable gyroscopic systems or statically unstable systems. Note that the identity inertia matrix indicates that normal modes discretization is applied in the formulation. All coefficient matrices are of dimension $n \times n$, the actuator influence matrix B is of $n \times m$, and the control force u is of $m \times 1$.

Although the requirement of direct velocity measurement may be restrictive in hardware implementation, colocated velocity feedback is the most widely discussed method in the structural control studies for its positive damping argumentation. Such control law makes the closed-loop system asymptotically stable with infinite gain margin. The system dynamics can be described by n modes with no residual modes, and the feedback controller is

$$u = -K_v B_v^T \dot{x} \tag{2}$$

where K_v is the velocity feedback control gain matrices, and B_v is the measurement sensor influence matrix. Substituting Eq. (2) into Eq. (1), the feedback control system becomes

$$\ddot{x} + (C + BK_v B_v^T)\dot{x} + \Lambda x = 0 \tag{3}$$

$$\ddot{x} + (D + G)\dot{x} + \Lambda x = 0 \tag{4}$$

where D and G are termed generalized damping and gyroscopic matrices, respectively.

$$D = C + 1/2\,(BK_v B_v^T + B_v K_v^T B^T)$$

$$G = 1/2\,(BK_v B_v^T + B_v K_v^T B^T)$$

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Usually a gyroscopic matrix appears in rotating structure systems, and it is a function of rotating speed. Note that $\boldsymbol{D}$ is symmetric while $\boldsymbol{G}$ is skew-symmetric. In the case of a colocated sensor and actuator pair with positive definite gain, the skew-symmetric matrix vanishes.

For state feedback control of flexible structures, the equation governing the closed-loop system dynamics remains in the form of a second-order equation as shown in Eq. (4) in terms of gyroscopic matrix as well as generalized inertia, damping, and stiffness matrices. Thus, stability prediction of a closed-loop system is similar to that of a discrete, linear dynamical system. Several techniques for investigating the stability of Eq. (4) are available. Stability criteria for second-order structure systems developed by Kliem and Pommer[6] and Ahmadian and Inman[7] can be applied. However, most criteria are based on the Lyapunov method, hence a method of generating the Lyapunov function is required. Lyapunov function exists but is rarely obtainable for systems with asymmetric coefficient matrices. Consequently, application of the stability criteria derived from the Lyapunov method is often very restrictive. In other related work, Piche[8] has proposed a symmetrizability criterion for stability check of structure systems with displacement feedback. Zhang and Shelly[9] have also derived two theorems, each predicting the stability of a SISO and a SIMO velocity feedback system. But the symmetrizability criterion and the stability theorems have little practical benefit in structural control design. Application is restrictive; direct computation of the closed-loop eigenvalues would be much easier. Most recently, Yang and Mote[10] have developed several criteria in predicting the stability of linear, second-order gyroscopic systems. Similar to their work, stability of the closed-loop system, Eq. (4), can be analyzed through its eigensolutions. Instability is predicted if at least one eigenvalue $\lambda = \alpha + j\beta$, $j = \sqrt{-1}$, satisfies $\alpha \geq 0$ and $\beta = 0$, giving divergence instability, or $\alpha > 0$ and $\beta \neq 0$, indicating flutter instability. Divergence is a static instability, and flutter, on the other hand, is a dynamic instability with increasing, oscillating amplitude. Consider the solution of Eq. (4) $\boldsymbol{x} = \boldsymbol{v}\, e^{\lambda t}$ where $\boldsymbol{v}$ and λ are the eigensolution. Define $k = \boldsymbol{v}^*\Lambda\boldsymbol{v}$, $d = \boldsymbol{v}^*\boldsymbol{D}\boldsymbol{v}$, and $g = -j\boldsymbol{v}^*\boldsymbol{G}\boldsymbol{v}$, where k, d, and g are real. The superscript * denotes a complex conjugate. The eigenvalues of Eq. (4) are the roots of

$$\lambda^2 + (d + jg)\lambda + k = 0 \tag{5}$$

A Routh-Hurwitz criterion can then be applied to derive the stability criteria.

For colocated feedback, $\boldsymbol{B} = \boldsymbol{B}_v$ and $\boldsymbol{G}=0$, the generalized damping matrix $\boldsymbol{D} = \boldsymbol{C} + \boldsymbol{B}\boldsymbol{K}_v\boldsymbol{B}^T$ is a positive definite if the control gain is a positive definite. Consequently, the closed-loop system is asymptotically stable, provided that $\Lambda > 0$. This stabilization by colocated velocity feedback has been studied previously by many researchers on nongyroscopic systems. Martin and Bryson[11] have stated that, for a single input colocated velocity feedback, the finite zeros are interlacing with the open-loop poles along an imaginary axis. The result is extended to a multi-input case by Williams.[12] Colocated feedback produces interlacing pole/zero patterns, hence the closed-loop system is inherently robust. However, the equation of a closed-loop system with noncolocated feedback contains not only the symmetric generalized damping but also the skew-symmetric gyroscopic matrix. Thus, structure control systems with noncolocated velocity feedback can be unstable; the destabilization occurs when the interlacing pole/zero pattern is destroyed. Physically the instability indicates an actuator and sensor pair located at the opposite side of a nodal point, nodal line, or nodal surface of the structure. A noncolocated sensor and actuator should then be positioned away from the nodal points, nodal lines, and nodal surfaces while maintaining the interlacing pole/zero pattern, thereby at minimum phase. But such placement may not be feasible when attempting to control systems with high modal density or systems with a large number of structural modes. Hence, two questions can be raised. First, given sensor location $\boldsymbol{B}_v$, how should one select the actuator location and design the control law such that a noncolocated feedback system remains stable and provides damping argumentation? Second, does a noncolocated feedback ever stabilize an otherwise unstable system?

The answer to the first question is similar to the stability prediction of a gyroscopic structure system. It can be shown from the roots of Eq. (5) that the system with $d > 0$ and $k > 0$ is asymptotically stable for all g, and it is unstable when $d < 0$ for all g. This phenomenon is because gyroscopic force never destabilizes an otherwise stable system, and never stabilizes an unstable damped system, either. If the noncolocated feedback leads to a positive definite generalized damping matrix, $\boldsymbol{D} > 0$, then the feedback system is stable regardless of its gyroscopic terms, which are also induced by velocity feedback. Conversely, for an unwise actuator placement $\boldsymbol{B}$ or an inadequate control gain matrix $\boldsymbol{K}_v$ leading to negative damping argumentation, $\boldsymbol{D} < 0$, the closed-loop system is destabilized.

To avoid such destabilization, a noncolocated velocity feedback system can be designed as follows. With the sensor location $\boldsymbol{B}_v$, place the actuator $\boldsymbol{B}$ at the same column space of $\boldsymbol{B}_v$ such that

$$\boldsymbol{B}^T = \boldsymbol{A}\boldsymbol{B}_v^T \tag{6}$$

where $\boldsymbol{A}$ is an $m \times m$ transformation matrix. The gain matrix is then selected as

$$\boldsymbol{K}_v = (\boldsymbol{L}\boldsymbol{L}^T + \boldsymbol{T})\boldsymbol{A} \tag{7}$$

where $\boldsymbol{L}$ is an arbitrary matrix and $\boldsymbol{T}$ is skew-symmetric, both of order $m \times m$. With the previous actuator placement and control law design, the noncolocated feedback system becomes

$$\ddot{\boldsymbol{x}} + (\boldsymbol{C} + \boldsymbol{B}\boldsymbol{L}\boldsymbol{L}^T\boldsymbol{B}^T + \boldsymbol{B}\boldsymbol{T}\boldsymbol{B}^T)\,\dot{\boldsymbol{x}} + \Lambda\boldsymbol{x} = 0 \tag{8}$$

Therefore, the generalized damping matrix $\boldsymbol{D}=(\boldsymbol{C}+\boldsymbol{B}\boldsymbol{L}\boldsymbol{L}^T\boldsymbol{B}^T)>0$ for any control gain $\boldsymbol{L}$ and $\boldsymbol{T}$ and the system with $\Lambda > 0$ is asymptotically stable with infinite gain margin. Note that for the same transformation matrix $\boldsymbol{A}$, there can be more than one solution of $\boldsymbol{B}$ satisfying Eq. (6). These multiple solutions indicate that one can move the actuator from one set of positions to the other and preserve the stability robustness property.

The control law in Eq. (7) provides a much greater latitude for tuning the natural frequencies of a closed-loop system. Should one prefer energy transfer between the vibration modes, the skew-symmetric part of the gain matrix $\boldsymbol{T}$ can provide such a transferring mechanism. For example, if the energy in the lower modes can be effectively transferred to the higher modes through $\boldsymbol{T}$, then the structural control performance of vibration suppression can be increased. Conversely, when $\boldsymbol{T} = 0$, the closed-loop system is similar to a damping system with little energy transferred between modes.

In the case of a single input system with positive gain, noncolocated velocity feedback, the closed-loop system is asymptotically stable only if $b_i v_{v_i} \leq 0$ where b_i and b_{vi} are the ith element of $\boldsymbol{B}$ and $\boldsymbol{B}_v$, respectively. Such a condition implies that if the sensor and actuator are in phase on a vibration mode, then this mode is asymptotically stable. If $b_i v_{v_i} < 0$; i.e., 180 deg out of phase, then the closed-loop system is unstable.

All of the previous conditions consider structure systems with positive definite stiffness matrix $\Lambda > 0$. However, a gyroscopic structure system can remain stable at some $\Lambda < 0$; a stable spinning top is one example. A gyroscopic system with $\Lambda < 0$ is normally referred to as being in a supercritical state. Yang and Mote[10] have shown that the sufficient and necessary condition for Eq. (4) to be stable is $\boldsymbol{D} = 0$ and $(4\Lambda - \boldsymbol{G}\boldsymbol{G}) > 0$. Application of the colocated velocity feedback leads to $\boldsymbol{D} > 0$, so that the closed-loop system is unstable. This is one unique case where colocated velocity feedback, contrary to common belief, destabilizes an otherwise stable system. Similarly, an unstable system ($\Lambda < 0$) can never be stabilized by colocated velocity feedback; a noncolocated pair is required for stabilization.

In conclusion, colocated velocity feedback on a nongyroscopic structure system is always constructive because of positive damping argumentation. But such colocation can destabilize a gyroscopic structure system. Stability criteria for systems with noncolocated velocity feedback are provided by Eqs. (6) and (7). The

Table 1 Structural control with velocity feedback, $\ddot{x} + C\dot{x} + \Lambda x = Bu$, $u = -K_v B_v^T \dot{x}$

	c/nc[a]	Condition	s/u[b]
1	c	$\Lambda > 0$, $K_v > 0$	s
2	$B = B_v$	$\Lambda < 0$, $K_v > 0$	u
3		$\Lambda > 0$, $C + 1/2(BK_vB_v^T + B_vK_v^TB^T) > 0$	s
4	nc	$\Lambda > 0$, $C + 1/2(BK_vB_v^T + B_vK_v^TB^T) < 0$	u
5		$\Lambda > 0$, $B^T = AB_v^T$, $K_v = LL^TA + T$, $T = -T^T$	s
6		$\Lambda > 0$, $B\ R^{n\times 1}$, $B = [b_i]$, $B_v = [b_{vi}]$, $b_i\, b_{vi} > 0$	s
7		$\Lambda < 0$, $C + 1/2(BK_vB_v^T + B_vK_v^TB^T) < 0$	u
8	$B \neq B_v$	$\Lambda < 0$, $C = 0$, $(4\Lambda - GG) > 0$ $G = 1/2(BK_vB_v^T + B_vK_v^TB^T)$	s
9		$\Lambda < 0$, $C = 0$, $(4\Lambda - GG) < 0$ $G = 1/2(BK_vB_v^T + B_vK_v^TB^T)$	u

[a]c/nc: colocated/noncolocated.
[b]s/u: stable/unstable

criteria guarantee a stable closed-loop system with infinite gain margin. One can achieve simultaneously both the damping argumentation and the energy transfer between vibration modes. Only the noncolocated feedback has the potential of stabilizing an unstable system. These constructive designs have not been reported as yet. The stability criteria are summarized in Table 1.

Conclusions

1) A stability robustness criterion of second-order structure systems with noncolocated velocity feedback is developed. Conditions are derived in terms of the structure inertia, damping, and stiffness matrices and the feedback control gain that guarantee the closed-loop stability.

2) Colocated velocity feedback on nongyroscopic structure systems is always constructive because of positive damping argumentation. The closed-loop system is asymptotically stable with infinite gain margin. Similarly, the closed-loop system with noncolocated feedback is also stable, provided that the symmetric part of the generalized damping matrix is positive definite. Stability robustness criteria are listed in Eqs. (6) and (7). One can achieve simultaneously both the damping argumentation and the energy transfer between vibration modes.

3) A gyroscopic structure system, e.g., a stable spinning top, can remain stable at some negative stiffness state, $\Lambda < 0$, but the application of colocated velocity feedback leads to an unstable system. This is one unique case where colocated velocity feedback, contrary to common belief, destabilizes an otherwise stable system. Similarly, an unstable system $\Lambda < 0$ can never be stabilized by colocated velocity feedback; a noncolocated pair is required for stabilization.

4) It is important to know that all the previous analysis shown, and hence the stability robustness criteria, are developed in the absence of actuator and sensor dynamics. Goh and Caughey[13] have indicated that second-order actuator dynamics can affect the stability robustness, provided that the actuator mass, damping, and stiffness properties are known. Spanos[14] also showed that sensor dynamics could limit the stability robustness. Further work on developing the stability robustness criteria for systems with both sensor and actuator dynamics is necessary.

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Nonlinear Response of Asymmetrically Laminated Plates in Cylindrical Bending

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I. Introduction

THE increasing use of high performance materials has intensified the demand for the development of tools to trace the response of laminated structures made by differently oriented laminas. Composite plates asymmetrically laminated (with respect to the middle surface of the laminate) are seldom used in structures, mainly because of the difficulty in controlling their configuration after curing. In reality, however, an asymmetric laminate may result from delamination, from surface damage in symmetric laminates, and from some geometrical imperfections in the thickness and/or in the orientation of each lamina. Since the pioneering studies made by Ambartsumyan[1] and Stavsky and Reissner[2] revealed the bending-stretching coupling effect in laminate composite plates, a considerable amount of work based on several theories and approximate methods of solution has been done on the nonlinear analysis of such plates. In fact, for the symmetric laminates, linear laminate plate theory is quite adequate if the transverse deflection is small compared with the plate thickness. But in an asymmetrically laminated plate, lateral deflection, i.e., plate bending, may be involved immediately when the load is applied, even though only in-plane loading exists. This early bending-extension coupling causes linear lamination theory to yield large errors in analyzing asymmetric laminates. As a consequence, their response should be computed

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